

Exercises: Mathematical Methods in Particle Physics

Series 1: Units, Quantities and Lorentz Invariance in Particle Physics

1. Complete the following table, in which the values of various physical quantities are given in the unit system of particle physics and in the SI system. In particle physics, a system of units is used where $\hbar = c = k_B = \epsilon_0 = 1$. The value G_{TP} of a quantity in particle physics can be converted to its value G_{SI} in SI units by multiplying G_{TP} by powers of the constants $\hbar$, c , k_B and ϵ_0 in the SI system such that G_{SI} has the correct units in the SI system:

$$G_{SI} = G_{TP} \cdot \hbar_{SI}^\alpha \cdot c_{SI}^\beta \cdot k_{B,SI}^\gamma \cdot \epsilon_{0,SI}^\delta \quad \text{for suitable } \alpha, \beta, \gamma, \delta.$$

Quantity	Value (particle physics)	Conversion (SI system)	Value (SI system)
Energy	1eV	$1\text{eV} := 1.602176634 \cdot 10^{-19}\text{J}$	$1.602176634 \cdot 10^{-19}\text{J}$
Length	1MeV^{-1}	$1\text{MeV}^{-1} \cdot \hbar c$	$1.973269804 \dots \cdot 10^{-13}\text{m}$
Time	1meV^{-1}		
Mass	1GeV		
Energy density	1GeV^4		
Temperature	1eV		
Electric charge	0.302822		
Electric field	1keV^2		
Electric potential	1keV	$1\text{keV}/\sqrt{\hbar c \epsilon_0}$	302.822V

2. In the Super Proton Synchrotron (SPS or Sp $\bar{\text{p}}$ SI) at CERN, protons and antiprotons were brought into collision collinearly. This made it possible for the first time to produce the heavy $W^\pm$ and Z bosons experimentally. The energy of both the protons and the antiprotons was 270GeV.

What energy did the antiprotons have in the rest frame of the protons immediately before the collision?

3. Let q be a timelike four-vector in the future light cone V^+ with $0 < q^2 = q_\mu q^\mu = m^2$. Furthermore, let $p = (p^0, \vec{p})$ also be a timelike four-vector in Minkowski space, with $p^2 > m^2$ and $p^0 > 0$. A scalar quantity t depending on p and q is given by $t = |\vec{q}'| \sqrt{p_0'^2 - m^2}$ in an inertial system IS' in which p has the special representation $p' = (p'^0, \vec{0})$ and q has the representation $q' = (q'^0, \vec{q}')$.

Give a Lorentz-invariant expression for t that contains only the Lorentz-invariant expressions $p^2 = p_\mu p^\mu$, q^2 and $pq = p_\mu q^\mu$. Hint: Consider the quantity $(pq)^2 - p^2 q^2$.

4. Field energy: In electrostatics, the Coulomb potential ($r = ||\vec{x}||$)

$$\Phi(\vec{x}) = \frac{q}{4\pi\epsilon_0 r}$$

of a point charge q is obtained as the solution of the Poisson equation that vanishes at infinity:

$$\Delta\Phi(\vec{x}) = -\frac{q}{\epsilon_0}\delta^{(3)}(\vec{x}).$$

In Newtonian gravitational theory, the Poisson equation for the gravitational potential $\tilde{\Phi}$ of a point mass m is analogously

$$\Delta\tilde{\Phi}(\vec{x}) = 4\pi Gm\delta^{(3)}(\vec{x}),$$

where the gravitational constant is given by $G = 6.67430(15) \cdot 10^{-11} \text{kg}^{-1}\text{s}^{-2}\text{m}^3$ and the gravitational acceleration, as a quantity analogous to the electric field, is given by $\vec{g}(\vec{x}) = -\vec{\nabla}\tilde{\Phi}(\vec{x})$.

Derive an expression for the energy density of the gravitational field analogous to the energy density $\epsilon_0\vec{E}^2/2$ of the electric field and calculate this energy density at the Earth's surface.

As is well known, electric charges of the same sign repel each other, whereas masses attract. Within the framework of a field-theoretic approach in flat spacetime, is the energy density of the Newtonian gravitational field positive or negative?