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Improved effective momentum approximation for quasielastic (e, e') scattering off highly charged nuclei

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Abstract. – We address the problem of including Coulomb distortion effects in inclusive quasielastic (e, e') reactions using an improved version of the effective momentum approximation. Arguments are given that a simple modification of the effective momentum approximation, which is no longer applicable in its original form in the case of highly charged nuclei, leads to very good results. A comparison of the improved effective momentum method with exact calculations is given. A critical remark concerns recent experiments with inclusive quasielastic positron scattering.

Nucleon knockout by quasielastic electron scattering provides a powerful possibility to explore the electromagnetic properties of nucleons and of the momentum distributions in nuclei, since the transparency of the nucleus with respect to the electromagnetic probe makes it possible to study the entire nuclear volume. Inclusive scattering provides information on a number of interesting nuclear properties. The width of the quasielastic peak allows a measurement of the nuclear Fermi momentum [1], whereas the tail of the quasielastic peak at low energy loss and large momentum transfer gives information on high-momentum components in nuclear wave functions [2]. The integral strength of quasielastic scattering, when compared to sum rules, gives information about the reaction mechanism and eventual modifications of nucleon form factors in the nuclear medium [3]. Finally, the scaling properties of the quasielastic response allow one to study the reaction mechanism [4], and extrapolation of the quasielastic response to $A = \infty$ provides us with a very valuable property of infinite nuclear matter [5].

The differential cross-section for the knockout process can be written in Born approximation as (for details see [6])

$$\frac{\mathrm{d}^4\sigma}{\mathrm{d}\epsilon_\mathrm{f}\,\mathrm{d}\Omega_\mathrm{f}\,\mathrm{d}E_\mathrm{f}\,\mathrm{d}\Omega_\mathrm{f}} = \frac{4\alpha^2\epsilon_\mathrm{f}^2 E_\mathrm{f} P_\mathrm{f}}{(2\pi)^5}\,\delta(\epsilon_\mathrm{i} + E_A - \epsilon_\mathrm{f} - E_\mathrm{f} - E_{A-1})\bar{\Sigma}|W_\mathrm{if}|^2, \tag{1}$$

where $\bar{\Sigma}$ indicates the sum (average) over final (initial) polarizations and

$$W_\mathrm{if} = \int \mathrm{d}^3x \int \mathrm{d}^3y \int \frac{\mathrm{d}^3q}{(2\pi)^2} j_\mu^\mathrm{e}(\vec{x}) \frac{e^{-i\vec{q}(\vec{x}-\vec{y})}}{q_\mu^2} J_\mathrm{N}^\mu(\vec{y}). \tag{2}$$

In this expression, $j^{\rm e}_{\mu}$ and $J^{\mu}_{\rm N}$ stand for the electron and nuclear currents, respectively. $\epsilon_{\rm i,f}$ ($E_{\rm i,f}$) are the initial and final energy of the electron (nucleon), and $P_{\rm f}$ is the final momentum of the nucleon. The δ-distribution in (1) assures energy conservation for the involved particles and the (residual) nucleus. The electron current is given by the well-known Dirac particle expression

$$j^{\mu}_{\rm e}(\vec{x}) = \bar{\Psi}^{\rm f}_{\rm e}(\vec{x})\gamma^{\mu}\Psi^{\rm i}_{\rm e}(\vec{x}). \tag{3}$$

For light nuclei, a description of electron wave functions by plane waves is a sufficient approximation for many applications, but for heavy nuclei Coulomb corrections (CC) may become large and affect the measured cross-sections; this needs to be accounted for, if one aims at a quantitative interpretation of data. Unfortunately, full distorted wave Born approximation calculations involving Dirac wave functions lead to an extensive calculational effort. Kim *et al.* proposed a local effective momentum approximation (LEMA), which leads to good results for heavy nuclei, but it still necessitates the introduction of non-planar wave functions [7].

The standard method in the case of light nuclei to handle CC for elastic scattering in the data analysis is the effective momentum approximation (EMA). The EMA accounts for two effects of the charged nucleus on the electron wave function. First, the initial and final electron momentum $\vec{k}_{\rm i,f}$ is enhanced in the vicinity of the nucleus due to the attractive electrostatic potential. Second, the attractive potential of the nucleus leads to a focusing of the electron wave function. For a highly relativistic electron with zero impact parameter the effective momenta $k'_{\rm i,f}$ of the electron in the center of the nucleus are given by

$$k'_{\rm i} = k_{\rm i} + \Delta k, \qquad k'_{\rm f} = k_{\rm f} + \Delta k, \qquad k_{\rm i,f} = \left|\vec{k}_{\rm i,f}\right|, \quad \Delta k = -V_0/c, \tag{4}$$

where V_0 is the potential energy of the electron in the center of the nucleus. The initial and final energy of the electron can be set equal to $\epsilon_{\rm i,f} = k_{\rm i,f}/c$. Kim *et al.* [7] calculated V_0 from the approximate formula

$$V_0 = -\frac{3\alpha Z}{2r_c}, \quad r_c = \left[1.1\,A^{1/3} + 0.86\,A^{-1/3}\right]\mathrm{fm}, \tag{5}$$

which is valid for heavy nuclei with charge Z and mass number A. *E.g.*, for ^{208}Pb we have $V_0 = -26.6\,\mathrm{MeV}$, not a negligible quantity when compared with energies of some hundreds of MeV typically used in electron scattering experiments.

Knoll [8] derived the enhancement factor $F_{\rm i,f}(\vec{r})$ of the electron wave amplitude in the vicinity of the nucleus from a high-energy partial wave expansion, following previous results given by Lenz and Rosenfelder [9,10]. The focusing factor in the center of the nucleus is given approximately by

$$F_{\rm i,f}(0) = k'_{\rm i,f}/k_{\rm i,f} = \left(1 - \frac{V_0}{\epsilon_{\rm i,f}}\right). \tag{6}$$

Therefore, EMA-corrected cross-sections are obtained by first calculating the cross-sections using plane electron waves but with the electron momenta replaced by the corresponding effective values. The result obtained this way must be multiplied by $F_{\rm i}^2$, since the focusing of the incoming wave enters quadratically in the cross-section. The focusing factor for the outgoing wave is automatically generated by the enhanced phase space factor $\sim {k'_{\rm f}}^2$ in the effective cross-section. Note that there is also an alternative but equivalent formulation of EMA [11]. There, the focusing factors are automatically absorbed in the Mott cross-section which is used as a prefactor in the full expression for the knockout cross-section. However, the matrix element for the knockout process is then defined without a photon propagator $\sim q_{\mu}^{-2}$.

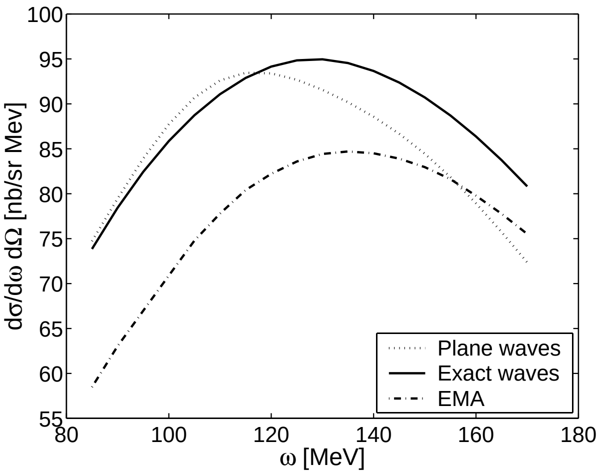

Fig. 1 – Cross-sections for inclusive (e, e') scattering on lead in different approaches (taken from [7], see fig. 4 therein) for initial electron energy $\epsilon_{\rm i} = 485\,{\rm MeV}$ and electron scattering angle $\Theta_{\rm e} = 60°$. Cross-sections obtained by Kim *et al.* from EMA (dash-dotted curve) deviate strongly from Kim's results obtained by using exact Dirac wave functions for electrons (solid curve). The dotted curve displays the cross-sections obtained from the plane-wave Born approximation, where the electron wave functions are described by simple Dirac plane waves.

Kim *et al.* [7] performed exact calculations for quasielastic electron scattering using Dirac wave functions both for electrons and nucleons. The calculations clearly show that the EMA has a tendency to underestimate the cross-sections in relevant kinematical regions which were explored experimentally at Saclay [12]. Figure 1 shows an example for initial electron energy $\epsilon_{\rm i} = 485\,{\rm MeV}$, scattering angle $\Theta_{\rm e} = 60°$ and varying electron energy loss ω.

But a simple modification of the effective momentum approximation, called EMA$'$ for short in this paper, significantly improves the situation. The key observation is the fact that most of the nucleons are located near the surface of the nucleus due to simple geometrical reasons, where the potential energy of the electron is given approximately by $2V_0/3$. But the focusing of the electron wave is described better by using the central potential value V_0. The reason for this is the fact that the focusing in the upstream side of the nucleus (with respect to the direction of the electron momentum) is smaller than in the center of the nucleus, but larger by a similar amount on the downstream side. Furthermore, the focusing does not fall off very strongly in transverse directions [13]. Therefore, a focusing factor

$$F_{\rm i,f} = \left(1 - \frac{V_0}{\epsilon_{\rm i,f}}\right) \tag{7}$$

is a good average value for the entire nuclear volume, but for the calculation of the effective cross-section,

$$k'_{\rm i} = k_{\rm i} + \Delta k', \qquad k'_{\rm f} = k_{\rm f} + \Delta k', \quad \Delta k' = -\frac{2}{3c} V_0 \tag{8}$$

is a better choice for the effective momenta. The EMA$'$ can therefore be expressed by the following simple recipe: First, determine the EMA-corrected cross-section, but with an effective potential value of $2V_0/3$. Second, multiply this cross-section by $[(k_{\rm i} + \Delta k)/(k_{\rm i} + \Delta k')]^2 [(k_{\rm f} + \Delta k)/(k_{\rm f} + \Delta k')]^2$, with Δk, $\Delta k'$ defined above, in order to account for the larger focusing according to the EMA$'$.

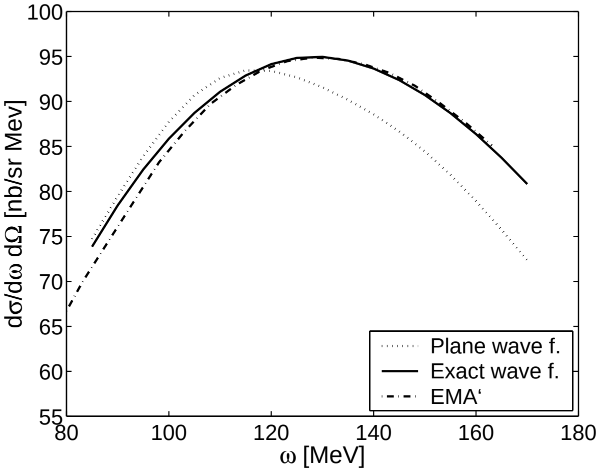

Fig. 2 – Modified effective momentum approximation: Approximate EMA′ values, derived from the EMA curve in fig. 1, are now in excellent agreement with exact values (the same solid line as in fig. 1).

As a first step, we checked our assumption using the results of Kim *et al.* [7], given here for initial electron energy $\epsilon_{\rm i} = 485\,{\rm MeV}$ and electron scattering angle $\Theta_{\rm e} = 60°$. The EMA results displayed in fig. 1 were obtained by Kim *et al.* from effective cross-sections with too large a $\Delta k = 26.6\,{\rm MeV}/c$ instead of using a $\Delta k' = (2/3) \cdot 26.6\,{\rm MeV}/c$. Our strategy was therefore to modify first these EMA values by correcting the focusing factor according to EMA′ values $\Delta k' = 26.6\,{\rm MeV}/c$ and $\Delta k = (3/2) \cdot 26.6\,{\rm MeV}/c$. The new values obtained this way correspond to potential values which are too large by 50 percent, but the focusing is then correct according to the EMA′ strategy. We then shifted the peak of the modified EMA curve towards the peak of the plane-wave curve by one third of the peak distance, such that we obtained a curve which gives a good estimate for the EMA′ values which would have been obtained by Kim *et al.* if they had applied the EMA′ method. The interpolation procedure used is only approximate, but we also used a different strategy, where we shifted the EMA curve first and applied then a correction for the focusing, with an equally good result. Since the Coulomb distortion is a correction and not excessively large, higher-order effects in the interpolation procedure do not play a crucial role. The result of the procedure is shown in fig. 2. EMA′ and exact values are in excellent agreement now. We also applied EMA′ to the values given in [7] for $\epsilon_{\rm i} = 310\,{\rm MeV}$ and electron scattering angle $\Theta_{\rm e} = 143°$, with an even better result in the region with low-energy transfer ω.

In conclusion, we present some calculations related to a recent experiment by Guèye *et al.* [14], where it was tried to give an experimental proof of the validity of the EMA also for inclusive quasielastic electron and positron scattering. The conclusion drawn by Guèye *et al.* is not in agreement with our results (see also [15]).

Guèye *et al.* measured positron quasielastic cross-sections on ^{12}C and ^{208}Pb and extracted the corresponding total response function which is defined in plane-wave Born approximation by

$$\frac{{\rm d}^2\sigma_{\rm PWBA}}{{\rm d}\Omega_{\rm f}\,{\rm d}\epsilon_{\rm f}} = \sigma_{\rm Mott} \times S^{\rm total}(|\vec{q}|, \omega, \Theta), \tag{9}$$

where

$$\sigma_{\rm Mott} = 4\alpha^2 \cos^2(\Theta/2)\epsilon_{\rm f}^2/q_\mu^4\,. \tag{10}$$

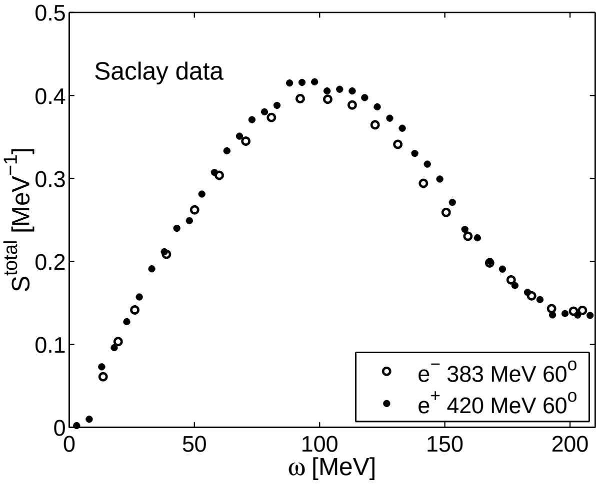

Fig. 3 – Typical experimental electron and positron response functions given by Guèye *et al.* for scattering off ^{208}Pb, which coincide after an additional minor rescaling of the electron data.

As mentioned above, the Mott cross-section remains unchanged when it gets multiplied by the EMA focusing factors and the momentum transfer q_μ^4 is replaced by its corresponding effective value, *i.e.* we have $F_\mathrm{i}^2 F_\mathrm{f}^2/q_{\mu,\mathrm{eff}}^4 = 1/q_\mu^4$, where $q_{\mu,\mathrm{eff}}^2 = \omega^2 - (\vec{k}_\mathrm{i}^\mathrm{eff} - \vec{k}_\mathrm{f}^\mathrm{eff})^2$. Note that $\epsilon_{\mathrm{i,f}}$ and the energy transfer ω are considered as fixed quantities [14].

Assuming that EMA is correct, it was found that the total response functions for electrons with an initial energy of 383 MeV and scattering angle 60° and positrons with an initial energy of 420 MeV and the same scattering angle are nearly identical (see fig. 3). This finding is based on the assumption that the relevant effective momenta are best described by a momentum shift of approximately $[(420-383)/2]\,\mathrm{MeV} \sim 18.5\,\mathrm{MeV}$ both for electrons and positrons, but with opposite sign. Whereas the electrons are accelerated to an effective momentum of approximately 401.5 MeV$/c$ in the relevant nuclear volume, the positron momentum is reduced to approximately the same value. But the coincidence of the response functions is obtained after a rescaling of the electron response, and the electron and positron data were obtained from different experiments.

In order to substantiate our observation concerning EMA′, we performed calculations using an eikonal approximation for electron and positron wave functions according to [16]. Since we used a single-particle shell model for the description of the ^{208}Pb nucleus, our model calculation does not contain contributions arising from correlations, meson-exchange currents or inelastic scattering from the nucleons in the nuclear ground state, but this does not affect the clear result which originates from the distortion of electron and positron wave functions due to the Coulomb field of the nucleus.

In our calculations we have chosen a slightly different initial energy of 385 MeV for electrons and 420 MeV for positrons, corresponding to a momentum shift from 385 MeV$/c$ to 402.5 MeV$/c$ for electrons and 420 MeV$/c$ to 402.5 MeV$/c$ for positrons. Using a simple EMA analysis, the response functions for e^+ and e^- obtained from the theoretical cross-sections differ by a considerable amount, as shown in fig. 4. But reducing the response function for electrons by a correction factor which accounts for the enhanced focusing of the electron wave function according to an EMA′ central potential value of -26 MeV and enhancing the response

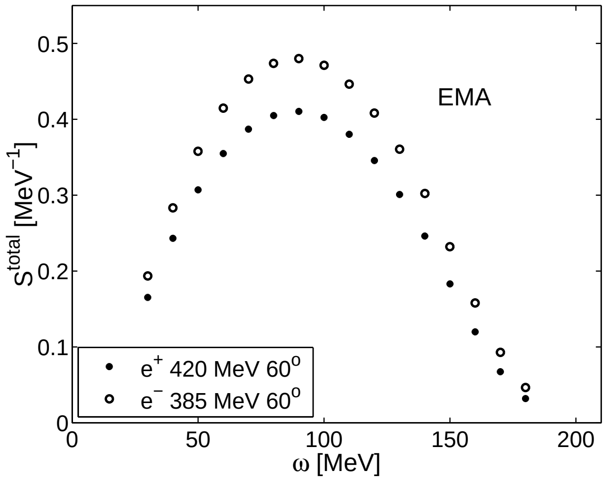

Fig. 4 – Theoretical response functions for electrons and positrons obtained from an EMA analysis for inclusive scattering off ^{208}Pb. The initial energy difference corresponds to a momentum shift $385\,\text{MeV}/c \rightarrow 402.5\,\text{MeV}/c$ for electrons and $420\,\text{MeV}/c \rightarrow 402.5\,\text{MeV}/c$ for positrons.

function for positrons according to a central potential value of 26 MeV leads to a satisfactory match of the two response functions, as shown in fig. 5.

We finally remark that the choice $\Delta k = -V_0/c$ and $\Delta k' = -2V_0/3c$ is an *ad hoc* prescription, but it is motivated from the physical picture that most of the nucleons of heavy nuclei are located near the surface of the nucleus. It clearly remains desirable to have access to exact calculations, but our calculations using the eikonal approximation for electron and positron wave functions and the exact calculations by Kim *et al.* clearly suggest that EMA is unreliable in the case of heavy nuclei, whereas EMA′ is a reliable approximation for the involved treatment of CC.

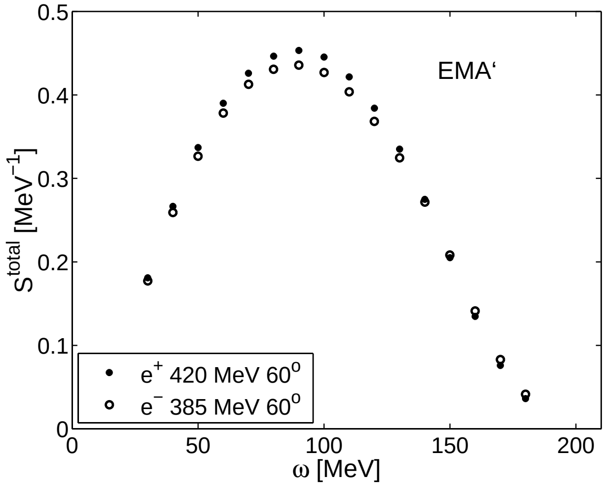

Fig. 5 – Theoretical response functions for electrons and positrons for scattering off ^{208}Pb, but EMA′ corrected.

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